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Electron transport in interacting hybrid mesoscopic systems

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Abstract. A unified theory for the current through a mesoscopic region of interacting electrons connected to two leads which can be either ferromagnet or superconductor is presented, yielding Meir-Wingreen-type formulas when applied to specific circumstances. In such a formulation, the requirement of gauge invariance is satisfied automatically. Moreover, one can judge unambiguously what quantities can be measured in the transport experiment.

PACS. 72.10.Bg General formulation of transport theory – 73.63.-b Electronic transport in nanoscale materials and structures – 72.25.-b Spin polarized transport – 74.50.+r Tunneling phenomena; point contacts, weak links, Josephson effects

Mesoscopic electron transport has received an increasing attention both theoretically and experimentally in last decade [1]. In mesoscopic or nanoscale systems the wave nature of electrons becomes apparent and the transport process is coherent. The Landauer-Büttiker formula [2], which encodes the current in the local properties of the interacting mesoscopic region and the equilibrium distribution functions of the noninteracting electron reservoirs, enhances our understanding of mesoscopic electron transport and has been applied successful in many fields [3]. In 1992 Meir and Wingreen [4] presented a formulation for electron transporting through a small confined region (quantum dot, QD) where the electron-electron interaction is important, and recovered the form of Landauer-Büttiker formula in the noninteracting case.

Recent advances in nanofabrication and material growth technologies make it possible to realize various kinds of hybrid mesoscopic structures [5–12], of which the building blocks are normal metals (N), ferromagnets (F) and superconductors (S). It is known that transport in the presence of a ferromagnet and a superconductor will be strongly related to the spin polarization of the ferromagnet and the Andreev reflection at the boundary of the superconductor [13,14]. The co-existence of two ferromagnets or two superconductors is revealed to display spin-valve effect [15] or Josephson effect [16]. When an interacting normal metal is connected with bulk ferromagnet(s) and/or superconductor(s), it is expected that the interplay among the electron-electron interaction, spin imbalance, Andreev reflection would induce more interesting or even more surprising features in mesoscopic electronic transport. Previous theoretical investigations on the transport properties of specific mesoscopic hybrid structures, such as N-QD-S [17], S-QD-S [18], F-QD-F [19], F-QD-S [20] etc., make some intuitive presumptions and hence lack mathematical rigidity, which will be discussed below in detail.

In this paper, we provide a scheme to treat the transport problem in an interacting hybrid mesoscopic structure in a unified way by using the Keldysh formalism [4, 21]. It is shown that gauge invariance can be satisfied automatically. It is also shown that what physical quantities can be measured in experiment.

We start with the Hamiltonian

$$\mathcal{H} = \mathcal{H}_{\mathcal{L}} + \mathcal{H}_{\mathcal{R}} + \mathcal{H}_{\mathcal{C}} + \mathcal{H}_{\mathcal{T}}, \tag{1}$$

where $\mathcal{H}_{\mathcal{C}} = \sum_{n\sigma}(\varepsilon_{n\sigma} - \mu_{\mathcal{C}})\psi^{\dagger}_{dn\sigma}\psi_{dn\sigma} + \mathcal{H}_{int}\left(\{\psi^{\dagger}_{dn\sigma}\}, \{\psi_{dn\sigma}\}\right)$ is the Hamiltonian for the central interaction region, $\mathcal{H}_{\mathcal{L}}$ ($\mathcal{H}_{\mathcal{R}}$) for the left (right) lead can be either the *Stoner* model [15] characterized by an exchange magnetization h with polar angle θ or the BCS Hamiltonian [16] with order parameter $\Delta = |\Delta|e^{i\varphi}$: $\mathcal{H}^{(F)}_{\gamma} = \sum_{k\sigma}[\varepsilon_{\gamma k\sigma} + \mathrm{sign}(\sigma)h_{\gamma}\cos\theta_{\gamma f} - \mu_{\gamma}]f^{\dagger}_{\gamma k\sigma}f_{\gamma k\sigma} + \sum_{k\sigma} h_{\gamma}\sin\theta_{\gamma f} f^{\dagger}_{\gamma k\sigma}f_{\gamma k\bar{\sigma}}$, or $\mathcal{H}^{(S)}_{\gamma} = \sum_{k\sigma}(\varepsilon_{\gamma k\sigma} - \mu_{\gamma})s^{\dagger}_{\gamma k\sigma}s_{\gamma k\sigma} + \sum_{k}\left[\Delta^{*}_{\gamma}s^{\dagger}_{\gamma k\uparrow}s^{\dagger}_{\gamma -k\downarrow} + \Delta_{\gamma}s_{\gamma -k\downarrow}s_{\gamma k\uparrow}\right]$, tunneling Hamiltonian can be written as

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$\mathcal{H}_{\mathcal{T}}^{\gamma(F)} = \sum_{kn;\sigma}\left[V_{kn;\sigma}^{\gamma f} f_{\gamma k\sigma}^{\dagger}\psi_{dn\sigma} + \text{h.c.}\right]$ or $\mathcal{H}_{\mathcal{T}}^{\gamma(S)} = \sum_{kn;\sigma}\left[V_{kn;\sigma}^{\gamma s} s_{\gamma k\sigma}^{\dagger}\psi_{dn\sigma} + \text{h.c.}\right]$. Here $\gamma = \mathcal{L}, \mathcal{R}$, $\mu_{\gamma/\mathcal{C}}$ is the chemical potential of the corresponding part, and hereafter the notations $\sigma = \uparrow, \downarrow$ and $\sigma = \pm$ are used interchangeably.

To see the tunneling processes more clearly, and even more importantly, to facilitate the analysis of the gauge invariance, and the simplification of the general current formula (9) to the form of specific systems, we first transform the *Stoner* and *BCS* Hamiltonian and the tunneling Hamiltonian by the following Bogoliubov transformations

$$f_{\gamma k\sigma} = \cos(\theta_{\gamma f}/2)\psi_{\gamma fk\sigma} - \text{sgn}(\sigma)\sin(\theta_{\gamma f}/2)\psi_{\gamma fk\bar{\sigma}},$$

$$\mathrm{e}^{-\mathrm{i}\varphi_\gamma/2}s_{\gamma k\sigma} = \cos\theta_{\gamma sk'}\psi_{\gamma sk'\sigma} + \text{sgn}(\sigma)\sin\theta_{\gamma s}\mathcal{P}\psi^{\dagger}_{\gamma sk'\bar{\sigma}},$$

where $\theta_{\gamma sk'} = \arctan[(\varepsilon_{\gamma k\sigma} + \sqrt{\varepsilon_{\gamma k\sigma}^2 + \Delta_\gamma^2})/(\varepsilon_{\gamma k\sigma} - \sqrt{\varepsilon_{\gamma k\sigma}^2 + \Delta_\gamma^2})]^{1/2}$ and $\mathcal{P}$ ($\mathcal{P}^\dagger$) is the pair destruction (creation) operator guaranteeing the particle conservation, transforming a state of a given N particles into that with $(N+2)/(N-2)$ particles, *i.e.*, $\mathcal{P}^\dagger/\mathcal{P}|N\rangle = |N+2\rangle/|N-2\rangle$, thus make sense the abnormal off-diagonal Green's functions consists of two creation or destruction particle operators. One finds that the lead Hamiltonian is diagonalized after the above Bogoliubov transformations.

In order to treat the ferromagnet and superconductor on the same footing, we introduce a 4-dimensional Nambu-spinor space, denoted by $\mathbf{\Psi}_\alpha = \left(\psi_{\alpha\uparrow}^\dagger\ \psi_{\alpha\downarrow}\ \psi_{\alpha\downarrow}^\dagger\ \psi_{\alpha\uparrow}\right)^\dagger$ and the Green's function in the Keldysh formalism [21] $\mathbf{G}_{\alpha,\beta}(t_1,t_2) = \mathrm{i}\langle T_C(\mathbf{\Psi}_\alpha(t_1)\otimes\mathbf{\Psi}_\beta^\dagger(t_2))\rangle$, where T_c is the contour-order operator, including $\mathbf{G}_{\alpha,\beta}^{r/a}(t_1,t_2) = \mp\mathrm{i}\vartheta(\pm t_1 \mp t_2)\langle\{\mathbf{\Psi}_\alpha(t_1),\mathbf{\Psi}_\beta^\dagger(t_2)\}_+\rangle$, and $\mathbf{G}_{\alpha,\beta}^{</>}(t_1,t_2) = \pm\mathrm{i}\langle\mathbf{\Psi}_\beta^\dagger(t_2)/\mathbf{\Psi}_\alpha(t_1)\otimes\mathbf{\Psi}_\alpha(t_1)/\mathbf{\Psi}_\beta^\dagger(t_2)\rangle$. The tunneling Hamiltonian in such a representation takes the form

$$\mathcal{H}_{\mathcal{T}}^{\gamma(F)} = \sum_{kn}\left(\mathbf{\Psi}_{\gamma fk}^\dagger\mathbf{V}_{kn}^{\gamma f}(t)\mathbf{\Psi}_{dn} + \text{h.c.}\right), \tag{2}$$

$$\mathcal{H}_{\mathcal{T}}^{\gamma(S)} = \sum_{kn}\left(\mathbf{\Psi}_{\gamma sk'}^\dagger\mathbf{V}_{kn}^{\gamma s}(t)\mathbf{\Psi}_{dn} + \text{h.c.}\right), \tag{3}$$

where $\mathbf{V}_{kn}^{\gamma f}(t) = \mathbf{R}^f(\frac{\theta_{\gamma f}}{2})\mathbf{V}_{kn}^{\gamma f}\mathbf{P}(\mu_{\gamma\mathcal{C}}t)$, $\mathbf{V}_{kn}^{\gamma s}(t) = \mathbf{R}^s(\theta_{\gamma sk'})\mathbf{V}_{kn}^{\gamma s}\mathbf{P}(\mu_{\gamma\mathcal{C}}t + \frac{\varphi_\gamma}{2})$, with $\mu_{\gamma C} = \mu_\gamma - \mu_{\mathcal{C}}$,

$$\mathbf{V}_{kn}^{\gamma f/s} = \begin{pmatrix} V_{kn}^{\gamma f/s} & 0 & 0 & 0 \\ 0 & -V_{kn}^{\gamma f/s*} & 0 & 0 \\ 0 & 0 & V_{kn}^{\gamma f/s} & 0 \\ 0 & 0 & 0 & -V_{kn}^{\gamma f/s*} \end{pmatrix},$$

and the unitary *rotation* and *phase* operators (matrices) are given by

$$\mathbf{R}^f(x) = \begin{pmatrix} \cos x & 0 & \sin x & 0 \\ 0 & \cos x & 0 & -\sin x \\ -\sin x & 0 & \cos x & 0 \\ 0 & \sin x & 0 & \cos x \end{pmatrix},$$

$$\mathbf{R}^s(x) = \begin{pmatrix} \cos x & -\mathcal{P}\sin x & 0 & 0 \\ \mathcal{P}^*\sin x & \cos x & 0 & 0 \\ 0 & 0 & \cos x & \mathcal{P}\sin x \\ 0 & 0 & -\mathcal{P}^*\sin x & \cos x \end{pmatrix},$$

$$\mathbf{P}(x) = \begin{pmatrix} e^{ix/\hbar} & 0 & 0 & 0 \\ 0 & \mathrm{e}^{-ix/\hbar} & 0 & 0 \\ 0 & 0 & \mathrm{e}^{ix/\hbar} & 0 \\ 0 & 0 & 0 & \mathrm{e}^{-ix/\hbar} \end{pmatrix}.$$

The chemical potential $\mu_{\gamma/\mathcal{C}}$ is incorporated into the *phase* matrix $\mathbf{P}$ and the ferromagnetism and superconductivity are reflected in the corresponding *rotation* matrices $\mathbf{R}^f$ and $\mathbf{R}^s$. The tunneling Hamiltonian now represents the explicit physical processes in the *semiconductor model* [16]: an electron of spin σ in the central regime can tunnel into either the spin σ band or $\bar{\sigma}$ band of the ferromagnetic lead, or tunnel into a spin σ state or condensate into an electron pair with a hole state of opposite spin being created; and *vice versa.* The total probability of the two tunneling processes into the same lead is $\cos^2 x + \sin^2 x = 1$. As shown below, these *rotation* and *phase* matrices are very useful in our analysis of gauge invariance, and even more importantly, the simplification of the formulas.

The current from the left lead into the interacting region is [4]

$$\begin{aligned}\mathcal{I}_{\mathcal{L}}(t) &= -e\langle\dot{N}_{\mathcal{L}}\rangle \\ &= \frac{2e}{\hbar}\text{Re}\sum_{nk}^{i=1,3}\left(\mathbf{V}_{kn}^{\gamma f/s\dagger}(t)\mathbf{G}_{\gamma f/sk,dn}^{<}(t,t)\right)_{ii} \\ &= \frac{2e}{\hbar}\text{Re}\sum_{nm}^{i=1,3}\int_{-\infty}^{t}\mathrm{d}t_1\Big(\mathbf{\Sigma}_{\mathcal{L}f/s;nm}^{r}(t,t_1)\mathbf{G}_{dm,dn}^{<}(t_1,t) \\ &\quad +\mathbf{\Sigma}_{\mathcal{L}f/s;nm}^{<}(t,t_1)\mathbf{G}_{dm,dn}^{a}(t_1,t)\Big)_{ii},\end{aligned} \tag{4}$$

where the self-energy matrices after converting the sum $\sum_k$ into an integral $\int \mathrm{d}\varepsilon_k\rho_{\sigma/N}^{\gamma f/s}(\varepsilon_k)$(where $\rho_{\sigma/N}^{\gamma f/s}$ is the

$$\mathbf{\Gamma}_x^{\gamma s}(\varepsilon \mp c) = \Gamma^{\gamma s}\begin{pmatrix} x^{\gamma s}(\varepsilon - c) & -\frac{|\Delta_\gamma|}{\varepsilon + c}x^{\gamma s}(\varepsilon + c) & 0 & 0 \\ -\frac{|\Delta_\gamma|}{\varepsilon - c}x^{\gamma s}(\varepsilon - c) & x^{\gamma s}(\varepsilon + c) & 0 & 0 \\ 0 & 0 & x^{\gamma s}(\varepsilon - c) & \frac{|\Delta_\gamma|}{\varepsilon + c}x^{\gamma s}(\varepsilon + c) \\ 0 & 0 & \frac{|\Delta_\gamma|}{\varepsilon - c}x^{\gamma s}(\varepsilon - c) & x^{\gamma s}(\varepsilon + c) \end{pmatrix},$$

$$\varrho^{\gamma s}(\varepsilon) = -\mathrm{i}\frac{\varepsilon\vartheta(|\Delta_\gamma| - |\varepsilon|)}{\sqrt{|\Delta_\gamma|^2 - \varepsilon^2}} + \frac{|\varepsilon|\vartheta(|\varepsilon| - |\Delta_\gamma|)}{\sqrt{\varepsilon^2 - |\Delta_\gamma|^2}},$$

$$\Gamma_{mn}^{\gamma s} = 2\pi\rho_N^{\gamma s}(0)V_{km}^{\gamma f\dagger}V_{kn}^{\gamma f}.$$

spin-dependent/normal density of states of the ferromagnet/superconductor) are

$$\mathbf{\Sigma}_{\gamma f;mn}^{r/a}(t_1,t_2) = \mp\frac{\mathrm{i}}{2}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{e}^{-\mathrm{i}\varepsilon(t_1-t_2)/\hbar}\mathbf{R}^{f\dagger}\left(\frac{\theta_{\gamma f}}{2}\right) \times\mathbf{\Gamma}_{mn}^{\gamma f}(\varepsilon \mp \mu_{\gamma\mathcal{C}})\mathbf{R}^f\left(\frac{\theta_{\gamma f}}{2}\right), \tag{5}$$

$$\mathbf{\Sigma}_{\gamma s;mn}^{r/a}(t_1,t_2) = \mp\frac{\mathrm{i}}{2}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{e}^{-\mathrm{i}\varepsilon(t_1-t_2)/\hbar}\mathbf{P}^{\dagger}\left(\mu_{\gamma\mathcal{C}}t_1 + \frac{\varphi_\gamma}{2}\right) \times\mathbf{\Gamma}_{\varrho/\varrho^*;mn}^{\gamma s}(\varepsilon \mp \mu_{\gamma\mathcal{C}})\mathbf{P}\left(\mu_{\gamma\mathcal{C}}t_1 + \frac{\varphi_\gamma}{2}\right), \tag{6}$$

$$\mathbf{\Sigma}_{\gamma f;nm}^{</>}(t_1,t_2) = \mathrm{i}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{e}^{-\mathrm{i}\varepsilon(t_1-t_2)/\hbar}\mathbf{R}^{f\dagger}\left(\frac{\theta_{\gamma f}}{2}\right) \times\mathbf{\Gamma}_{nm}^{\gamma f}(\varepsilon \mp \mu_{\gamma\mathcal{C}})\,\mathbf{R}^f\left(\frac{\theta_{\gamma f}}{2}\right)\left[\mathbf{f}_\gamma(\varepsilon \mp \mu_{\gamma\mathcal{C}}) - \frac{1}{2}\mathbf{1} \pm \frac{1}{2}\mathbf{1}\right], \tag{7}$$

$$\mathbf{\Sigma}_{\gamma s;nm}^{</>}(t_1,t_2) = \mathrm{i}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{e}^{-\mathrm{i}\varepsilon(t_1-t_2)/\hbar}\mathbf{P}^{\dagger}\left(\mu_{\gamma\mathcal{C}}t_1 + \frac{\varphi_\gamma}{2}\right) \times\mathbf{\Gamma}_{\rho;nm}^{\gamma s}(\varepsilon \mp \mu_{\gamma\mathcal{C}})\left[\mathbf{f}_\gamma(\varepsilon \mp \mu_{\gamma\mathcal{C}}) - \frac{1}{2}\mathbf{1} \pm \frac{1}{2}\mathbf{1}\right] \times\mathbf{P}\left(\mu_{\gamma\mathcal{C}}t_1 + \frac{\varphi_\gamma}{2}\right), \tag{8}$$

with (subscripts mn is omitted in the matrices)

$$\mathbf{f}_\gamma(\varepsilon \mp c) = \begin{pmatrix} f(\varepsilon - c) & 0 & 0 & 0 \\ 0 & f(\varepsilon + c) & 0 & 0 \\ 0 & 0 & f(\varepsilon - c) & 0 \\ 0 & 0 & 0 & f(\varepsilon + c) \end{pmatrix},$$

$$\mathbf{\Gamma}^{\gamma f}(\varepsilon \mp c) = \begin{pmatrix} \Gamma_\uparrow^{\gamma f}(\varepsilon - c) & 0 & 0 & 0 \\ 0 & \Gamma_\downarrow^{\gamma f}(\varepsilon + c) & 0 & 0 \\ 0 & 0 & \Gamma_\downarrow^{\gamma f}(\varepsilon - c) & 0 \\ 0 & 0 & 0 & \Gamma_\uparrow^{\gamma f}(\varepsilon + c) \end{pmatrix},$$

$$\Gamma_{\sigma;mn}^{\gamma f}(\varepsilon) = 2\pi\rho_\sigma^{\gamma f}(\varepsilon)V_{km}^{\gamma f\dagger}V_{kn}^{\gamma f},$$

and ($x = \varrho, \varrho^*, \rho$)

see equation above

Here $f(x) = 1/(1 + \mathrm{e}^x/k_BT)$ is the Fermi distribution function. The quasi-particle density of states of the *BCS* superconductor is $\rho^{\gamma s}(\varepsilon) = \mathrm{Re}\{\varrho^{\gamma s}(\varepsilon)\}$.

The current flowing from the right lead $\mathcal{I}_\mathcal{R}$ can be obtained in a similar way. In the steady transport problem, the current is uniform if no charge piles up in the central region, that is, $\mathcal{I}_\mathcal{L} = -\mathcal{I}_\mathcal{R}$. Symmetrizing equation (4) one finds

$$\mathcal{I}(t) = \frac{e}{\hbar}\mathrm{Re}\sum^{i=1,3}\int_{-\infty}^{t}\mathrm{d}t_1\mathrm{Tr}\Big\{\big[\mathbf{\Sigma}_{\mathcal{L}f/s}^r(t,t_1) - \mathbf{\Sigma}_{\mathcal{R}f/s}^r(t,t_1)\big] \times\mathbf{G}_{d,d}^<(t_1,t) + \big[\mathbf{\Sigma}_{\mathcal{L}f/s}^<(t,t_1) - \mathbf{\Sigma}_{\mathcal{R}f/s}^<(t,t_1)\big]\mathbf{G}_{d,d}^a(t_1,t)\Big\}_{ii}, \tag{9}$$

where the trace is over the level indices in the central region. Equation (9) with the self-energy matrices (5, 6, 7, 8) is the central result of this work, it expresses the current through an interacting region in terms of the local properties of such region ($\mathbf{G}_{d,d}^{r,a/<}$) and the equilibrium distribution functions ($\mathbf{\Sigma}_{\gamma f/s}^{r,a/<}$) of the attached leads, as in the work of Meir and Wingreen [4]. It can be applied to many types of hybrid mesoscopic structures even in the non-equilibrium situation, allowing various kinds of interactions in the central region. Notice that, the current is generally time-independent, except the case of two superconducting leads with nonzero bias, and that the full Green's functions $\mathbf{G}_{d,d}^{r,a/<}$ should be evaluated with the consideration of the tunneling between the interacting region and the leads. The retarded/advanced Green's functions $\mathbf{G}_{d,d}^{r,a}$ can be calculated in several approaches, such as the equation of motion formalism [22], interpolative method [23] and the NCA technique [24]. While the lesser one $\mathbf{G}_{d,d}^<$ can be obtained from the Keldysh's equation based on Ng's ansatz [25]. With the help of the unitary property of the *phase* operator $\mathbf{P}$, it is not difficult to find that equation (9) is gauge invariant, namely, $\mathcal{I}(t)$ remains unchanged under a global energy shift. When the bias $V = (\mu_\mathcal{L} - \mu_\mathcal{R})/e$ becomes zero, one can readily find that $\mathcal{I}(t) = 0$ except the case of two superconducting leads with different superconducting phases, due to the coherent transport of quasi-particle pairs [26].

In the following, we apply this generalized result equation (9) to the specific structures we are interested in. When the two leads are both ferromagnetic, after a *phase* and a *rotation* transformation, equation (9) reduces to

$$\mathcal{I}_{fnf} = \frac{\mathrm{i}e}{2\hbar}\sum^{i=1,3}\int\frac{\mathrm{d}\varepsilon}{2\pi}Tr\Big\{\Big(\big[\hat{\mathbf{\Gamma}}^{\mathcal{L}f}(\varepsilon\mp eV)-\mathbf{\Gamma}^{\mathcal{R}f}(\varepsilon)\big] \times\widehat{\mathbf{G}}^{<}_{d,d}(\varepsilon)+\big[\hat{\mathbf{\Gamma}}^{\mathcal{L}f}(\varepsilon\mp eV)\mathbf{f}_{\mathcal{L}}(\varepsilon\mp eV) -\mathbf{\Gamma}^{\mathcal{R}f}(\varepsilon)\mathbf{f}_{\mathcal{R}}(\varepsilon)\big]\big[\widehat{\mathbf{G}}^{r}_{d,d}(\varepsilon)-\widehat{\mathbf{G}}^{a}_{d,d}(\varepsilon)\big]\Big)_{ii}\Big\}, \quad (10)$$

where $\hat{\mathbf{\Gamma}}^{\mathcal{L}f}=\mathbf{R}^{f\dagger}(\frac{\theta_f}{2})\mathbf{\Gamma}^{\mathcal{L}f}\mathbf{R}^{f}(\frac{\theta_f}{2})$, $\theta_f=\theta_{\mathcal{L}f}-\theta_{\mathcal{R}f}$, and

$$\widehat{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon)=\int\mathrm{d}(t-t')\mathrm{e}^{i\varepsilon(t-t')/\hbar}\mathbf{P}(\mu_{\mathcal{RC}}t)\mathbf{R}^{f}(\frac{\theta_{\mathcal{R}f}}{2}) \times\mathbf{G}^{r,a/<}_{d,d}(t,t')\mathbf{R}^{f\dagger}(\frac{\theta_{\mathcal{R}f}}{2})\mathbf{P}^{\dagger}(\mu_{\mathcal{RC}}t').$$

One sees that equation (10) is formally the same as the Meir-Wingree formula [4] in the normal lead case. The current is time-independent as one might expect, and just depends on the relative angle θ_f between two magnetization orientations and the bias V, the difference between the chemical potential of the two leads.

The current through an interaction region with a ferromagnetic and a superconducting lead can be derived similarly

$$\mathcal{I}_{fns} = \frac{\mathrm{i}e}{2\hbar}\sum^{i=1,3}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{Tr}\Big\{\Big(\big[\mathbf{\Gamma}^{\mathcal{L}f}(\varepsilon\mp eV)-\mathbf{\Gamma}^{\mathcal{R}s}_{\rho}(\varepsilon)\big] \times\overline{\mathbf{G}}^{<}_{d,d}(\varepsilon)+\big[\mathbf{\Gamma}^{\mathcal{L}f}(\varepsilon\mp eV)\mathbf{f}_{\mathcal{L}}(\varepsilon\mp eV) -\mathbf{\Gamma}^{\mathcal{R}s}_{\rho}(\varepsilon)\mathbf{f}_{\mathcal{R}}(\varepsilon)\big]\big[\overline{\mathbf{G}}^{r}_{d,d}(\varepsilon)-\overline{\mathbf{G}}^{a}_{d,d}(\varepsilon)\big]\Big)_{ii}\Big\}, \quad (11)$$

in which the full Green's functions are

$$\overline{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon)=\int\mathrm{d}(t-t')\mathrm{e}^{i\varepsilon(t-t')/\hbar}\mathbf{P}\left(\mu_{\mathcal{RC}}t+\frac{\varphi_{\mathcal{R}}}{2}\right) \times\mathbf{R}^{f}\left(\frac{\theta_{\mathcal{L}f}}{2}\right)\mathbf{G}^{r,a/<}_{d,d}(t,t')\mathbf{R}^{f\dagger}\left(\frac{\theta_{\mathcal{L}f}}{2}\right)\mathbf{P}^{\dagger}\left(\mu_{\mathcal{RC}}t'+\frac{\varphi_{\mathcal{R}}}{2}\right)\cdot$$

The current through a generic hybrid mesoscopic structure also depends on just the bias V. The dependence on the magnetization orientation of the ferromagnetic lead and the phase of the order parameter of the superconductor lead is absent after the *rotation* and *phase* operations, which are just presumptions in previous investigations [17–20].

When the two leads are both superconductors, the situation becomes complicated. However, we still obtain an elegant formula in this case following the same procedure as in the above derivation

$$\mathcal{I}_{sns}(t) = -\frac{e}{\hbar}\sum^{i=1,3}\int\frac{\mathrm{d}\varepsilon}{2\pi}\mathrm{ImTr}\Big\{\Big(\frac{1}{2}\big[\tilde{\mathbf{\Gamma}}^{\mathcal{L}s}_{\varrho}(\varepsilon\mp eV;t) -\mathbf{\Gamma}^{\mathcal{R}s}_{\varrho}(\varepsilon)\big]\widetilde{\mathbf{G}}^{<}_{d,d}(\varepsilon;t)-\big[\tilde{\mathbf{\Gamma}}^{\mathcal{L}s}_{\rho}(\varepsilon\mp eV;t) \mathbf{f}_{\mathcal{L}}(\varepsilon\mp eV)-\mathbf{\Gamma}^{\mathcal{R}s}_{\rho}(\varepsilon)\mathbf{f}_{\mathcal{R}}(\varepsilon)\big]\widetilde{\mathbf{G}}^{a}_{d,d}(\varepsilon;t)\Big)_{ii}\Big\}, \quad (12)$$

where

$$\tilde{\mathbf{\Gamma}}^{\mathcal{L}s}_{\varrho/\rho}(\varepsilon\mp eV;t)=\mathbf{P}^{\dagger}\left(eVt+\frac{\varphi_s}{2}\right)\mathbf{\Gamma}^{\mathcal{L}s}_{\varrho/\rho}(\varepsilon\mp eV) \times\mathbf{P}\left(eVt+\frac{\varphi_s}{2}\right),$$

$$\widetilde{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon;t)=\int\mathrm{d}(t-t')\mathrm{e}^{\mathrm{i}\varepsilon(t-t')/\hbar}\mathbf{P}\left(\mu_{\mathcal{RC}}t+\frac{\varphi_{\mathcal{R}}}{2}\right) \times\mathbf{G}^{r,a/<}(t,t')\mathbf{P}^{\dagger}\left(\mu_{\mathcal{RC}}t'+\frac{\varphi_{\mathcal{R}}}{2}\right),$$

with $\varphi_s=\varphi_{\mathcal{L}}-\varphi_{\mathcal{R}}$. Here we have added the time variable t into the full Green's function $\widetilde{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon;t)$, other than $\widehat{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon)$. The reason is that the full Green's functions $\widehat{\mathbf{G}}^{r,a/<}_{d,d}$ should be calculated in the presence of tunneling as well as interactions in the central region. In the present case, the t-dependence can not be avoided in the self-energy matrices (6, 8), while it can be removed by a unitary *phase* operation in the case of only one superconductor lead. The current through an interacting mesoscopic region with two superconductor leads, is generally time dependent, as in the case of weak Josephson links [16]. However, in the limiting case of zero bias, the current is a time-independent nonzero quantity, as can be found from equation (12).

The time-dependence of $\widehat{\mathbf{G}}^{r,a/<}_{d,d}$ arises from the couplings to the voltage biased two superconducting leads. One can show that $\widehat{\mathbf{G}}^{r,a/<}_{d,d}$ depends only on the single time variable t at least within perturbative analysis. In fact the Green's function $\widehat{\mathbf{G}}^{r,a/<}_{d,d}$ can be expanded in powers of the fundamental frequency $\omega_0=2eV/\hbar$, *i.e.*, $\widehat{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon,t)=\sum_m\widehat{\mathbf{G}}^{r,a/<}_{d,d}(\varepsilon,\varepsilon+m\omega_0/2)\mathrm{e}^{\mathrm{i}m\omega_0t/2}$, which with the expression for the Green's function $\widehat{\mathbf{G}}^{r,a/<}_{d,d}$ below equation (12) is exactly the form of the double-energy transformation [27]. And the current can be generally expressed as $\mathcal{I}(t)=\sum_n I_n\mathrm{e}^{in\omega_0t}$.

The ferromagnetic or superconductor lead will be in a normal state, when the magnetization h or the order parameter Δ becomes zero. Such an observation allows us to study a more broad category of the hybrid structures using the above formalism. By defining the unitary operators, we prove rigorously that the *relative* value between some quantities, such as chemical potential, magnetization orientation, and order parameter phase etc. can be measured. *All these quantities can be expressed as the energy-independent arguments of the exponential functions or the triangle functions in a unitary matrix*, reflecting some kind of requirement of the symmetrical invariance. As a further example, we recover all the known formulas obtained in the absence of interactions [17–20,26]. This proves the validity and generality of our formalism in other respects. Taking into consideration the electron-electron interactions, one expect rich physics to show up, especially in the Kondo regime.

In summary, we have given a unified formula for the current through an interacting region with either superconducting or ferromagnetic leads. The current formula derived satisfies gauge invariance automatically. Such a current formula can be applied to an appreciable class of hybrid mesoscopic systems, allowing arbitrary interactions within the central nanoscale region.

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